NOTATION

$\bar{\alpha}$, Average heat-transfer coefficient; F, area of heat-transfer surface of duct; $\Delta\bar{t}_{log}$, log-mean temperature difference; Q, quantity of heat; $\bar{t}_w$, average wall temperature; t_1, t_{i+1}, thermocouple readings; l_i, thermocouple spacing; k, number of thermocouples; $\overline{Nu}_0$, ξ_0, average Nusselt number and coefficient of hydraulic friction for smooth duct; $\overline{Nu}$, ξ, the same for the duct with intensifier; Re, Reynolds number; α, flow swirl angle; μ, effective viscosity of fluid; C_p, specific heat at constant pressure; λ, thermal conductivity; D, inside diameter of duct; d, diameter of central stem of intensifier; ρ, density of fluid; l, length of duct.

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COUPLED PROBLEM OF STEADY-STATE HEAT TRANSFER DURING TURBULENT FLOW OF LIQUID THROUGH A PLANE SLOT WITH DISSIPATION OF MECHANICAL ENERGY

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An analytical solution is obtained for the coupled problem of steady-state heat transfer during turbulent flow of a liquid through a plane slot with dissipation of mechanical energy.

The problem of steady-state heat transfer during turbulent flow of the coolant and the need to account for dissipation of mechanical energy in the analysis of this problem arise in practical applications [1] such as transport of crude oil under conditions of Northern climate [2].

We make the following assumptions: 1) The flow of the fluid and the heat transfer are both quasisteady; 2) the fluid is incompressible and its physical properties remain constant; 3) the change in the thermal flux density along the axis caused by heat conduction and turbulent heat transfer is small in comparison with its change in the transverse direction [1]; 4) the flow through the heat-transfer region has been hydrodynamically stabilized; 5) the temperature of the liquid and the slot wall in the entrance section is a known function of the transverse coordinate y; 6) the temperature of the outside surface of the slot is given as an arbitrary integrable function of the axial coordinate x.

The problem will be formulated as a system of equations in dimensionless variables: equation of energy for the liquid

$$W(Y)\,\frac{\partial\Theta_1}{\partial X}=\frac{\partial}{\partial Y}\left[\frac{\mathrm{Re}}{2\xi_0}\left(\frac{1}{\mathrm{Pr}}+\frac{\varepsilon_\sigma}{\nu}\right)\frac{\partial\Theta_1}{\partial Y}\right]+\frac{\mathrm{Br}}{\mathrm{Pr}}\sqrt{\frac{\xi}{8}}\left(1+\frac{\varepsilon_\sigma}{\nu}\right)\left(\frac{dW}{dY}\right)^2,\quad 0<X<\infty,\ 0<Y<1, \tag{1}$$

equation of heat conduction for the wall

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$$\frac{4}{\mathrm{Re}^2}\frac{\partial^2\Theta_2}{\partial X^2}+\frac{\partial^2\Theta_2}{\partial Y^2}=0,$$
$$0<X<\infty,\ 1<Y<\delta, \tag{2}$$

with the boundary conditions

$$\Theta_1(0,\ Y)=f_1(Y), \tag{3}$$

$$\frac{\partial\Theta_1(X,\ 0)}{\partial Y}=0, \tag{4}$$

$$\Theta_2(0,\ Y)=f_2(Y), \tag{5}$$

$$\Theta_2(X,\ \delta)=\varphi(X), \tag{6}$$

$$\left.\frac{\partial\Theta_1}{\partial X}\right|_{X=\infty}=\left.\frac{\partial\Theta_2}{\partial X}\right|_{X=\infty}=0; \tag{7}$$

and the conditions of coupling for the temperature fields

$$\frac{\partial\Theta_1(X,\ 1)}{\partial Y}=K_\lambda\frac{\partial\Theta_2(X,\ 1)}{\partial Y},\ \Theta_1(X,\ 1)=\Theta_2(X,\ 1)=\chi(X). \tag{8}$$

We let function $\varphi(X)$ reach a finite value at $X\to\infty$ and let the unknown function at the coupling boundary be the temperature at that boundary, assuming also that $N_{Pr,turb}=1$.

For channels of simple geometrical shapes there are several semiempirical relations available which describe the velocity profile and yield the coefficients of turbulent momentum transfer. In this study will be used the Pey relations for the velocity profile W(Y) and the coefficient of turbulent momentum transfer ε_σ/ν.

The problem (1)-(8) can be solved by the method of finite differences [4, 5] or an exact analytical solution can be obtained [6], but the approximate analytical solution proposed here is more convenient for an analysis of coupled heat-transfer processes and engineering calculations.

The solution to the system of Eqs. (1)-(8) will be sought as the sum of two functions

$$\Theta_1(X,\ Y)=\Theta_{1\mathrm{W}}(Y)+\Theta_{1X}(X,\ Y), \tag{9}$$

$$\Theta_2(X,\ Y)=\Theta_{2\mathrm{W}}(Y)+\Theta_{2X}(X,\ Y). \tag{10}$$

For determining these functions we obtain from Eqs. (9) and (10) four interdependent equations with corresponding boundary conditions

$$\frac{d}{dY}\left[\frac{N_{Re}}{2\xi_0}\left(\frac{1}{\mathrm{Pr}}+\frac{\varepsilon_\sigma}{\nu}\right)\frac{\partial\Theta_{1\mathrm{W}}}{\partial Y}\right]+\frac{N_{Br}}{N_{Pr}}\sqrt{\frac{\xi}{8}}\left(1+\frac{\varepsilon_\sigma}{\nu}\right)\left(\frac{dW}{dY}\right)^2=0,\quad \frac{d\Theta_{1\mathrm{W}}(0)}{dY}=0; \tag{11}$$

$$\frac{d^2\Theta_{2\mathrm{W}}}{dY^2}=0, \tag{12}$$

$$\Theta_{2\mathrm{W}}(\delta)=\varphi(\infty),\ \Theta_{1\mathrm{W}}(1)=\Theta_{2\mathrm{W}}(1)=\chi_\infty,\ \frac{\partial\Theta_{1\mathrm{W}}(1)}{\partial Y}=K_\lambda\frac{\partial\Theta_{2\mathrm{W}}(1)}{\partial Y};$$
$$W(Y)\frac{\partial\Theta_{1X}}{\partial X}=\frac{\partial}{\partial Y}\left[\frac{N_{Re}}{2\xi_0}\left(\frac{1}{\mathrm{Pr}}+\frac{\varepsilon_\sigma}{\nu}\right)\frac{\partial\Theta_{1X}}{\partial Y}\right], \tag{13}$$

$$\Theta_{1X}(0,\ Y)=f_1-\Theta_{1\mathrm{W}}(Y),\quad \frac{\partial\Theta_{1X}(X,\ 0)}{\partial Y}=0;$$
$$\frac{4}{N_{Re}^2}\frac{\partial^2\Theta_{2X}}{\partial X^2}+\frac{\partial^2\Theta_{2X}}{\partial Y^2}=0,$$
$$\Theta_{2X}(0,\ Y)=f_2-\Theta_{2\mathrm{W}}(Y),\ \Theta_{2X}(X,\ \delta)=\varphi(X)-\Theta_{2\mathrm{W}}(Y), \tag{14}$$
$$\Theta_{1X}(X,\ 1)=\Theta_{2X}(X,\ 1)=\chi(X)-\chi_\infty=\chi_1(X),$$

$$\frac{\partial \Theta_{1X}(X,\ 1)}{\partial Y} = K_\lambda \frac{\partial \Theta_{2X}(X,\ 1)}{\partial Y}.$$

Integrating Eqs. (11) and (12) yields

$$\Theta_{1w}(Y) = c_1 + c_2 \int \frac{\partial Y}{\frac{N_{Re}}{2\xi_0}\left(\frac{1}{N_{Pr}} + \frac{\varepsilon_\sigma}{\nu}\right)} - \int \frac{\frac{2\xi_0}{N_{Re}} \sqrt{\frac{\xi}{8}} \frac{N_{Br}}{N_{Pr}} \int \left(1 + \frac{\varepsilon_\sigma}{\nu}\right)\left(\frac{dW}{dY}\right)^2 dY}{\left(\frac{1}{N_{Pr}} + \frac{\varepsilon_\sigma}{\nu}\right)} dY,$$

$$\Theta_{2w}(Y) = c_3 + c_4 Y.$$

The integration constants are determined from the conditions of coupling and the boundary conditions, with functions ε_σ/ν and W(Y) given.

Equations (13) and (14) were solved by a procedure involving concurrent application of the Laplace integral transformation and the orthogonal Bubnov–Galerkin method [7, 8]. Application of the Laplace integral transformation with respect to coordinate X yields in the transform domain

$$pW(Y)\overline{\Theta}_{1X}(p,\ Y) - W(Y)(f_1 - \Theta_{1w}(Y)) = \frac{\mathrm{Re}}{2\xi_0} \frac{\partial}{\partial Y}\left[\left(\frac{1}{N_{Pr}} + \frac{\varepsilon_\sigma}{\nu}\right)\frac{\partial \overline{\Theta}_{1X}(p,\ Y)}{\partial Y}\right],$$

$$\frac{\partial \overline{\Theta}_{1X}(p,\ 0)}{\partial Y} = 0,\ \overline{\Theta}_{1X}(p,\ 1) = \overline{\chi}_1(p),$$

$$\frac{4}{N_{Re}^2}[p^2\overline{\Theta}_{2X}(p,\ Y) - p(f_2 - \Theta_{2w}(Y))] -$$

$$-\frac{4}{N_{Re}^2} \left.\frac{\partial \Theta_{2X}(X,\ Y)}{\partial X}\right|_{X=0} + \frac{d^2\overline{\Theta}_{2X}(p,\ Y)}{dY^2} = 0,$$

$$\overline{\Theta}_{2X}(p,\ 1) = \overline{\chi}_1(p),\ \overline{\Theta}_{2X}(p,\ \delta) = \overline{\varphi}(p) - \overline{\varphi}(\infty),$$

$$\frac{\partial \overline{\Theta}_{1X}(p,\ 1)}{\partial Y} = K_\lambda \frac{\partial \overline{\Theta}_{2X}(p,\ 1)}{\partial Y}.$$

For the solution in the transform domain, the Bubnov–Galerkin method yields

$$\overline{\Theta}_{1X}(p,\ Y) = \overline{\chi}_1(p) + \sum_{k=1}^{N} \overline{a}_k(p)\, g_k(Y),$$

$$\overline{\Theta}_{2X}(p,\ Y) = \overline{\chi}_1 \frac{\delta - Y}{\delta - 1} + (\overline{\varphi}(p) - \Theta_{2w}(Y))\frac{Y - 1}{\delta - 1} + \sum_{k=1}^{M} \overline{t}_k(p)\,\Psi_k(Y).$$

The transform coefficients $\overline{a}_k(p)$ and $\overline{t}_k(p)$ are determined from the condition of orthogonality for the discrepancy in all coordinate functions $g_k(Y)$ and $\Psi_k(Y)$. For $\overline{a}_k(p)$ and $\overline{t}_k(p)$ we obtain the two systems of algebraic equations of orders N and M respectively

$$\sum_{k=1}^{N} (A_{kj} + pB_{kj})\,\overline{a}_k(p) = p\overline{\chi}_1(p)\,E_j + G_j, \tag{15}$$

$$\sum_{k=1}^{M} (a_{kj} + p^2 b_{kj})\,\overline{t}_k(p) = l_j p^2 \overline{\chi}_1(p) + \xi_j p + \nu_j, \tag{16}$$

where

$$A_{kj} = \int_0^1 \frac{\partial}{\partial Y}\left[\left(\frac{1}{\mathrm{Pr}} + \frac{\varepsilon_\sigma}{\nu}\right)\frac{\partial g_k}{\partial Y}\right] g_j(Y)\,dY;$$

$$B_{kj} = -\int_0^1 W(Y)\,g_k(Y)\,g_j(Y)\,dY;$$

$$E_j = \int_0^1 W(Y)\,g_j(Y)\,dY;\ G_j = -\int_0^1 W(Y)\,(f_1 - \Theta_{1w}(Y))\,g_j(Y);$$

$$a_{kj}=\int_1^\delta \frac{\partial}{\partial Y}\left(\frac{\partial \Psi_k(Y)}{\partial Y}\right)\Psi_j(Y)\,dY;\quad b_{kj}=\frac{4}{\mathrm{Re}^2}\int_1^\delta \Psi_k(Y)\,\Psi_j(Y)\,dY;$$

$$l_j=-\frac{4}{N_{\mathrm{Re}}^2}\int_1^\delta \frac{\delta-Y}{\delta-1}\Psi_j(Y)\,dY;\quad \xi_j=\frac{4}{N_{\mathrm{Re}}^2}\int_1^\delta (f_2-\Theta_{2w}(Y))\Psi_j(Y)dY;$$

$$\nu_j=\frac{4}{N_{\mathrm{Re}}^2}\int_1^\delta \left.\frac{\partial\Theta_2(X,\ Y)}{\partial X}\right|_{X=0}\Psi_j(Y)\,dY.$$

The coordinate functions $g_k(Y)$ are selected in the form

$$g_k(Y)=W(Y)\,Y^{2(k-1)},\ k=1,\ 2,\ \dots\ N.$$

The functions $\Psi_k(Y)$ are found from the solution to the corresponding Sturm–Liouville problem

$$\Psi_k(Y)=\sin k\pi\frac{Y-1}{\delta-1},\ k=1,\ 2,\ \dots\ M.$$

The solution to system (15), (16) is obtained according to Cramer's rule

$$\bar{a}_k(p)=\frac{\Delta_{1k}(p)}{\Delta(p)}\,p\bar{\chi}_1(p)+\frac{\Delta_{2k}(p)}{\Delta(p)},\ k=1,\ 2,\ \dots\ N,$$

$$\bar{t}_k(p)=\frac{D_{1k}(p)}{D(p)}\,p^2\bar{\chi}_1(p)+\frac{D_{2k}(p)}{D(p)}+\frac{D_{3k}(p)}{D(p)},\ k=1,\ 2,\ \dots\ N,$$

where $\Delta(p)$ and $\Delta_{jk}(p)$ are respectively the principal determinant and auxiliary determinants of system (15), $D(p)$ and $D_{nk}(p)$ ($n=1,\ 2,\ 3$) are respectively the principal determinant and auxiliary determinants of system (16). Function $\vec{\chi}_1(p)$ is found from the condition of coupling.

Using the expansion theorem for inverse transformation, we obtain the solution to problems (13) and (14)

$$\Theta_1(X,\ Y)=\chi_1(X)+\sum_{k=1}^{N}\left\{\int_0^X \chi_1'(t)\sum_j\frac{\Delta_{1k}(p_j)}{\Delta'(p_j)}\exp[p_j(X-t)]\,dt\right.$$
$$\left.+\sum_j\frac{\Delta_{2k}(p_j)}{\Delta'(p_j)}\exp(p_jX)\right\}g_k(Y),$$

$$\Theta_2(X,\ Y)=\chi_1(X)\frac{\delta-Y}{\delta-1}+\sum_{k=1}^{M}\left\{\int_0^X \chi_1'(t)\sum_j\frac{p_j^2D_{1k}(p_j)}{D'(p_j)}\exp[p_j(X-t)]dt\right.$$
$$\left.+\sum_j\frac{p_jD_{2k}(p_j)+D_{3k}(p_j)}{D'(p_j)}\exp(p_jX)\right\}\Psi_k(Y).$$

With the aid of expressions (9) and (10), we then obtain the solution to the boundary-value problem (1)-(8).

Using the thus obtained analytical solutions, one can calculate the mean-mass temperature Θ_m of the liquid as well as the Nusselt number and other characteristics of the heat transfer. The mean-mass temperature and the general Nusset number were indeed calculated numerically in this way, by the Bubnov–Galerkin method in the third-order approximation with a constant temperature of the wall and the liquid in the channel entrance section [$f_1=f_2=0$, $\varphi(X)=1$]. These calculations indicate that sufficiently reliable results can thus be obtained for the characteristics of heat transfer when $\delta<2$. The changing of the mean-mass temperature of the liquid during cooling, with $N_{\mathrm{Re}}=10^4$ and $N_{\mathrm{Pr}}=0.7$, is shown in Fig. 1. The graph indicates that the cooling process can be slowed down appreciably by an increase of the wall thickness and a decrease of its thermal conductivity. The variation of the general Nusselt number [9] along the channel, with $\delta=1.005$ and $K_\lambda=1000$, during cooling and during heating of the liquid is shown in Fig. 2a and b respectively. The maximum value of the Nusselt number, $N_{\mathrm{Nu}}=25.6$ when $N_{\mathrm{Br}}=0$, agrees with the result of another study [10].

The results of this study suggest that dissipation of mechanical energy can greatly increase the local values of the heat-transfer coefficient. Such an increase is attributable, first of all, to a radical restructurization

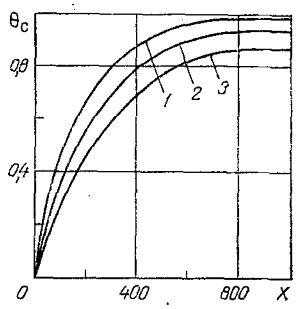

Fig. 1. Variation of the mean-mass temperature of the liquid ($N_{Re} = 10^4$, $N_{Pr} = 0.7$): 1) $N_{Br} = 0$, $\delta = 1.005$, $K_\lambda = 1000$; 2) $N_{Br} = -0.1$, $\delta = 1.005$, $K_\lambda = 1000$; 3) $N_{Br} = -0.1$, $\delta = 1.2$, $K_\lambda = 20$; $f_1 = f_2 = 0$; $\varphi(x) = 1$.

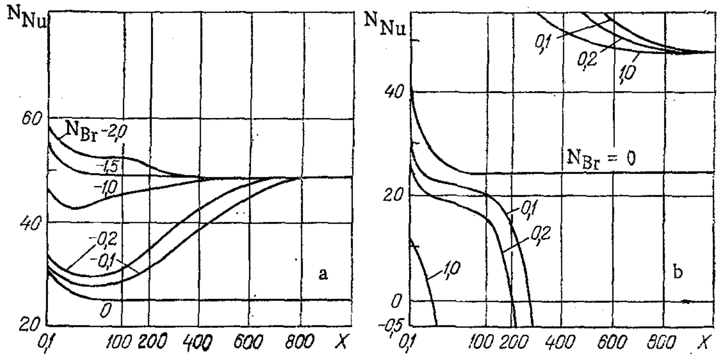

Fig. 2. Variation of the Nusselt number along the channel with thin wall when (a) $N_{Br} < 0$ and (b) $N_{Br} > 0$: $N_{Re} = 10^4$; $N_{Pr} = 0.7$; $\delta = 1.005$; $K_\lambda = 1000$; $f_1 = f_2 = 0$; $\varphi(x) = 1$.

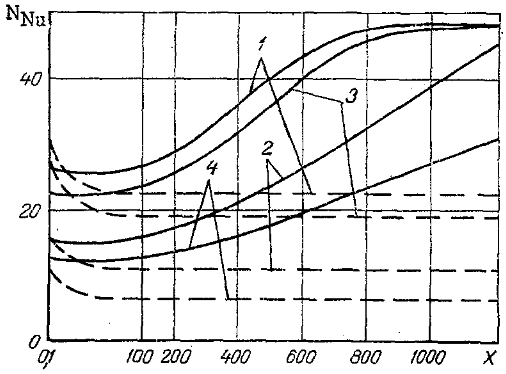

Fig. 3. Variation of the Nusselt number along the channel with energy dissipation as well as channel dimensions and thermophysical properties of channel wall and of liquid taken into account.

of the temperature profile in connection with the sharp increase of the temperature gradients at the channel walls, where mechanical energy is dissipated with particularly high intensity. During heating this dissipation of energy can result in reversals of the thermal flux (negative values of the Nusselt number, physically meaningless).

The dependence of the heat-transfer intensity on the thermophysical properties of the wall and of the liquid as well as on the channel dimensions is shown in Fig. 3. A comparison of the data here with those in Fig. 2a reveals that decreasing the thickness of the channel wall and decreasing the ratio K_λ will appreciably decrease the length of the initial thermal channel segment and decrease the Nusselt number within this segment. The results of these calculations and the thus obtained analytical solution to the coupled problem of convective heat transfer can be utilized for studying the role of dissipation processes in the heat transfer during turbulent flow of a liquid through a plane slot and the dependence of that heat transfer on the thickness of the slot wall and on the properties of the wall material.

NOTATION

$\Theta_1 = (t_1 - t_0)/(t_m - t_0)$, $\Theta_2 = (t_2 - t_0)/(t_m - t_0)$, Dimensionless temperature of the liquid and dimensionless temperature of the wall, respectively; x, axial coordinate; y, transverse coordinate; $X = 4x/N_{Re}2h$ and $Y = h/y$, dimensionless coordinates; $v_* = \bar{w}(\xi/8)^{1/2}$, dynamic velocity; $\bar{w}$, section velocity of the liquid; ξ, friction coefficient; 2h, inside slot dimension; $2h_1$, outside slot dimension; ν, kinematic viscosity; ε_σ, eddy viscosity; μ, dynamic viscosity; $W(Y) = w_x/v_*$, dimensionless velocity profile; w_x, axial projection of the velocity vector; λ_i (i = 1, 2), thermal conductivity of the liquid and of the wall material respectively; a, thermal diffusivity of the liquid; $N_{Re} = \bar{w}2h/\nu$, Reynolds number; $N_{Pr} = \nu/a$, Prandtl number; $N_{Br} = \mu\bar{w}^2/\lambda_1(t_m - t_0)$, Brinkman number; $\xi_0 = hv_*/\nu$; $\delta = h_1/h$; $K_\lambda = \lambda_2/\lambda_1$; t_0, temperature in the entrance section of the heat-transfer segment; t_m, temperature of the external surface of the tube wall on the heat exchanger surface; Θ_{1W}, Θ_{2W}, solutions to problems (11) and (12); and p, variable of Laplace integral transformation.

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